# Karnaugh Maps

*Rab Nawaz Khan Jadoon*

Lecturer

**COMSATS Lahore**

**Pakistan**

Department of Computer Science

**DCS**

COMSATS Institute of
Information Technology

**Digital Logic and Computer Design**

# Karnaugh Map

- A Karnaugh map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression.
- A Kamaugh map is similar to a truth table because it presents all of the possible values of input variables and the resulting output for each value.

# K-Maps

- Karnaugh map is an array of cells in which each cell represents a binary value of the input variables.

- The cells are managed in a way so that simplification of a given expression is simply a matter of properly grouping the cells.

- Karnaugh maps can be used for expressions with two, three, four. and five variables, but we will discuss only 3-variable and 4-variable situations to illustrate the principles.

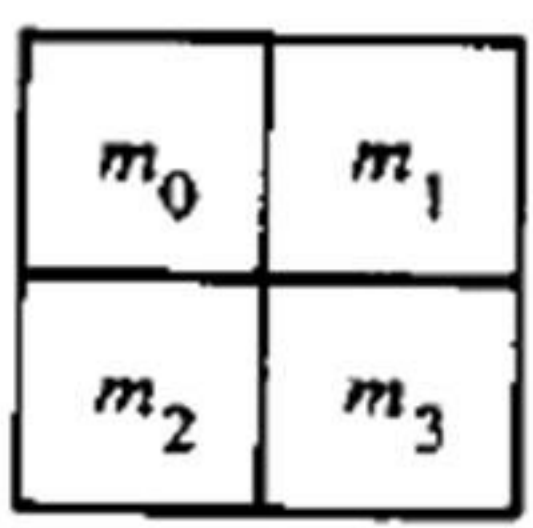
# K-Maps

- The number of cells in a Karnaugh map is equal to the total number of possible input variable combinations as is the number of rows in a truth table.

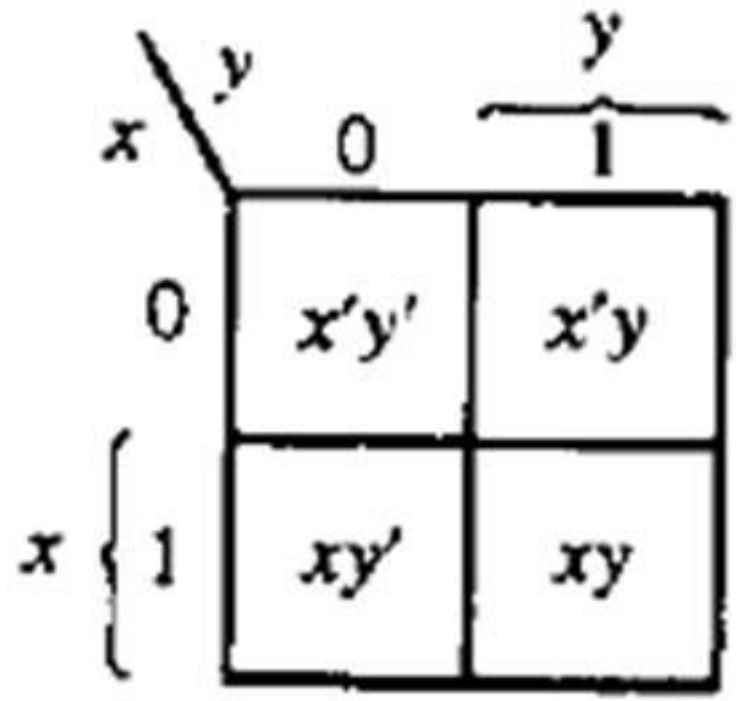
  - For two variables, the number of cells is $2^2 = 4$.
  - For three variables, the number of cells is $2^3 = 8$.
  - For four variables, the number of cells are $2^4 = 16$.

# 2 variable K map

- A two variable has four minterms, hence it has 4 squares one for each term, as shown below,




(a)




(b)

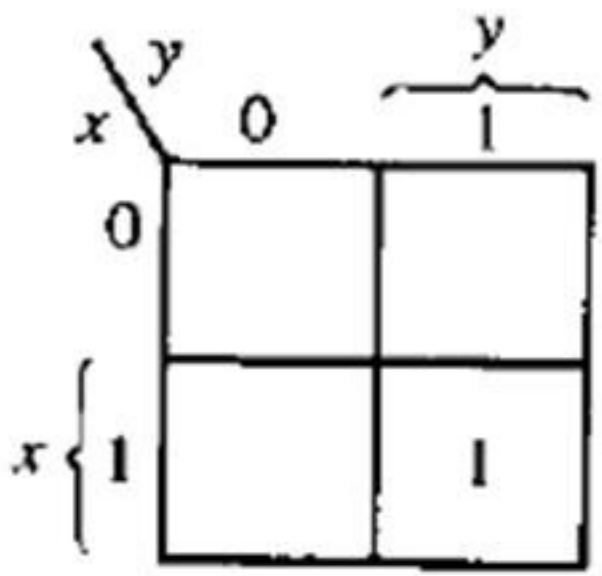
# Example

- Example: map the following functions into two veriable k-map,

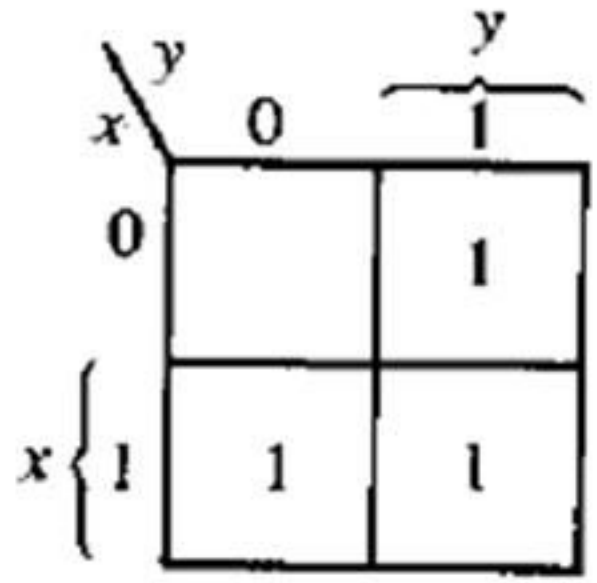
  **a. xy**

  **b. x + y= x′ y + xy′ + xy = $m_1 + m_2 + m_3$**

**Solution**




(a) $xy$

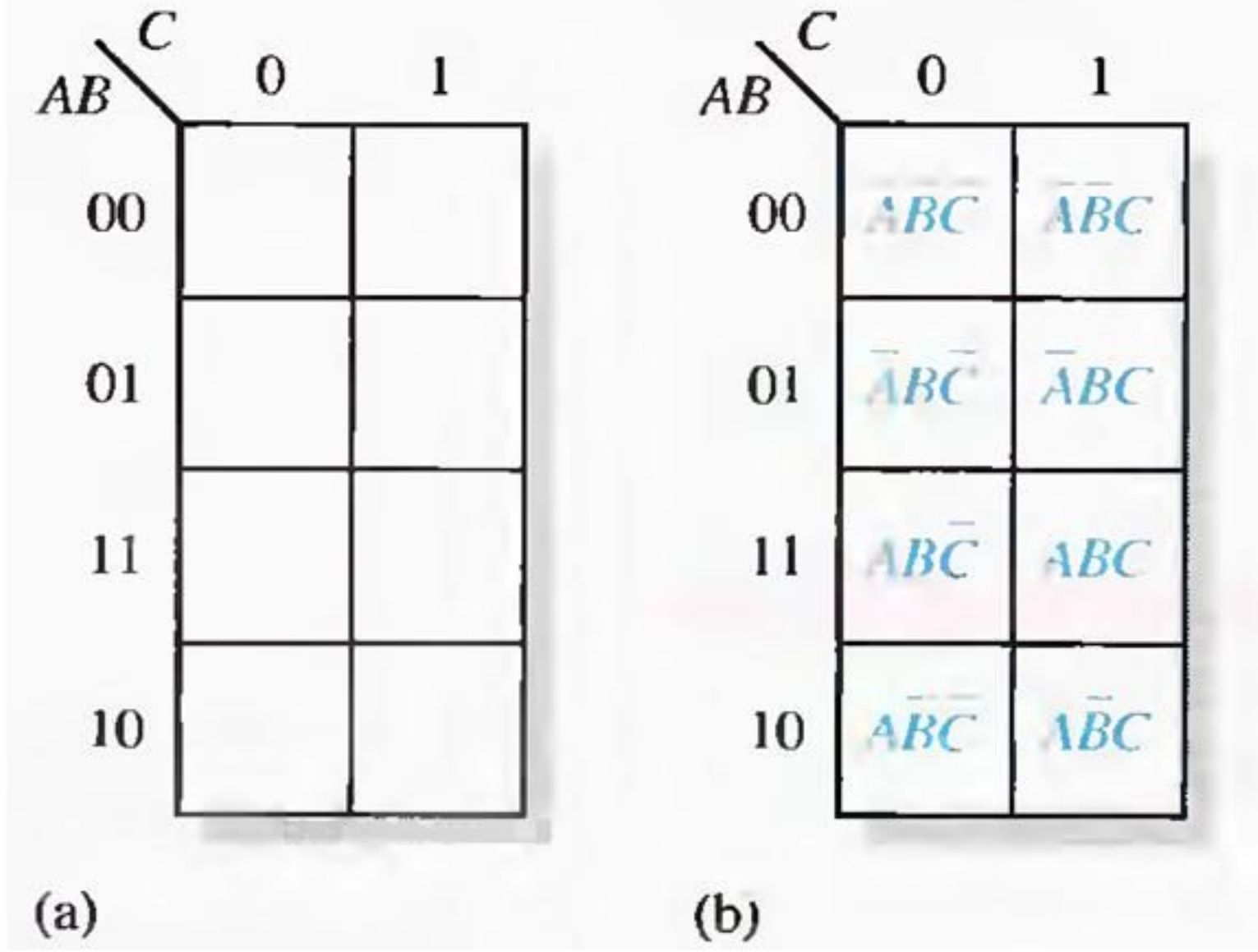



(b) $x + y$

- **The 3-Variable Karnaugh Map**
  - The 3-variable Karnaugh map is an array of eight cells.
  - In this case, A, B, and C are used for the variables although other letters could be used.
  - Binary values of A and B are along the left side (notice the sequence) and the values of C are across the top.

# K-Maps

- The value of a given cell is the binary values of A and B at the left in the same row combined with the value of C at the top in the same column.
- For example, the cell in the upper left corner has a binary value of 000 and the cell in the lower right corner has a binary value of 101.

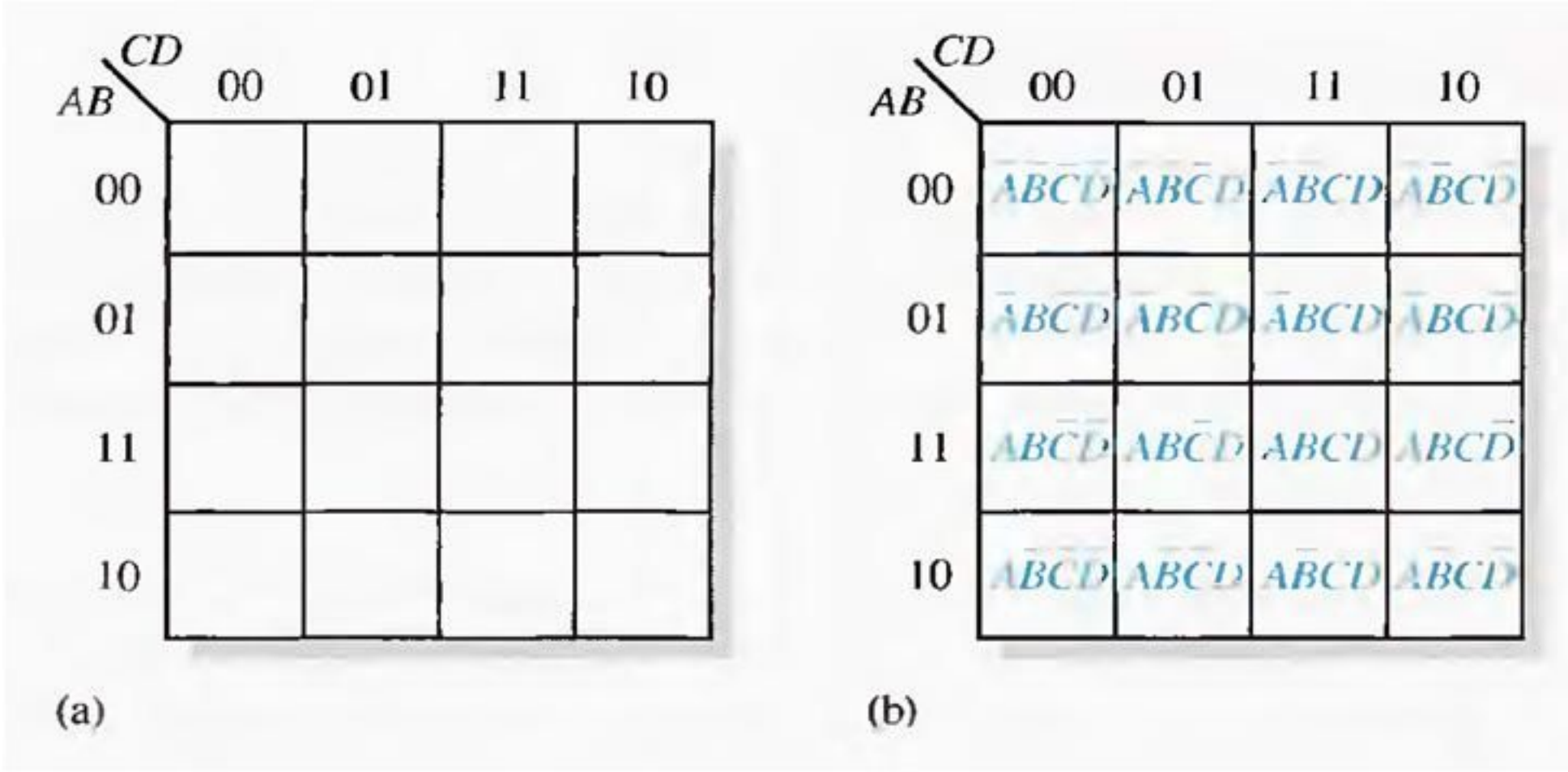



**3 variable K-Map**

# The 4-Variable Karnaugh Map

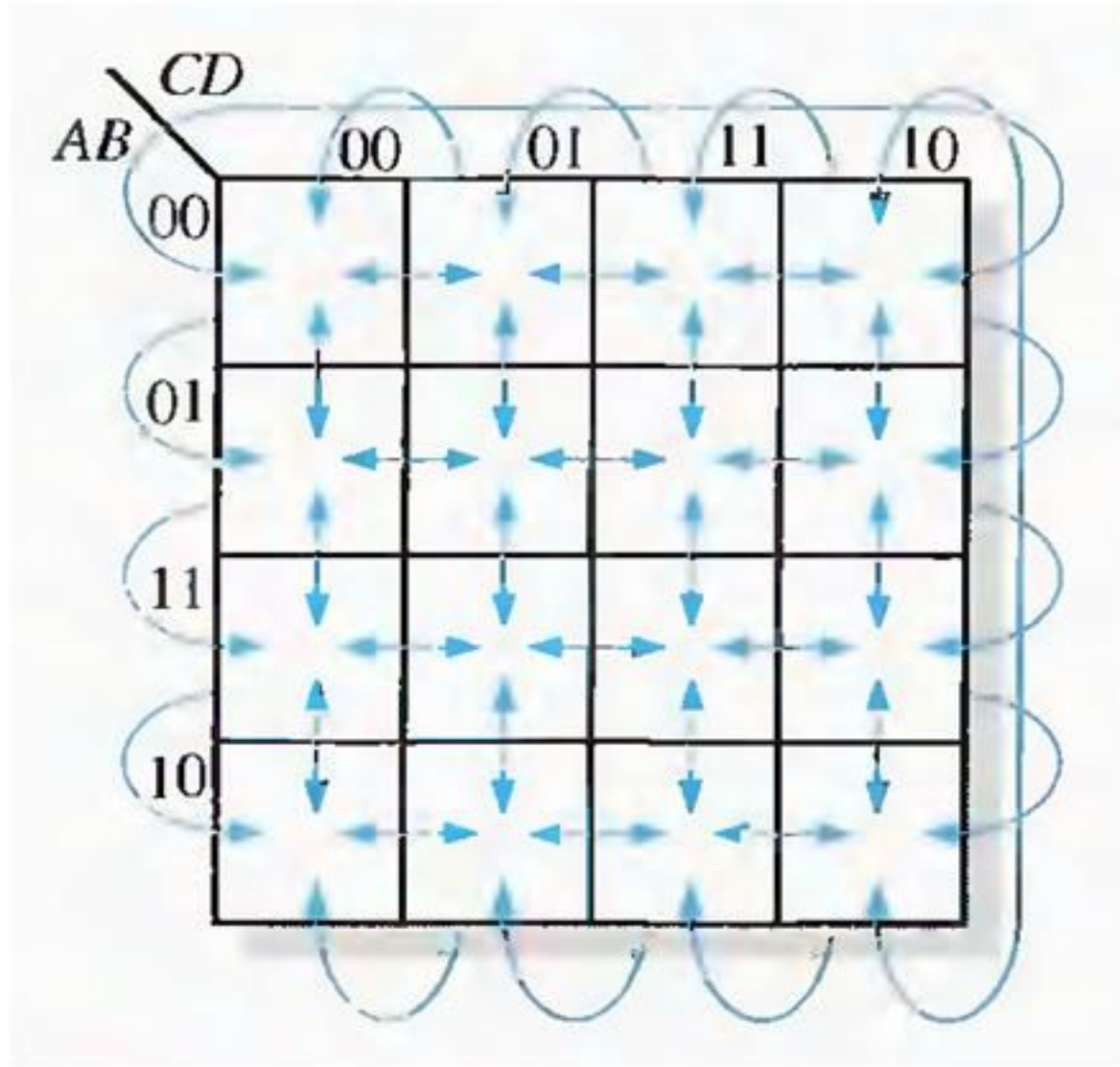
- The 4-Variable Karnaugh Map
  - The 4-variable Karnaugh map is an array of sixteen cells.
  - Binary values of A and B are along the left side and the values of C and D are across the top.
  - The value of a given cell is the binary values of A and B at the left in the same row combined with the binary values of C and D at the top in the same column.

- **Four veriable map**
  - For example, the cell in the upper right corner has a binary value of 00 I 0 and the cell in the lower right corner has a binary value of 1010.



- **Cell Adjacency**
  - The cells in a Karnaugh map are arranged so that there is only a single-variable change between adjacent cells.
  - Adjacency is defined by a single-variable change.
  - Cells with values that differ by more than one variable are not adjacent.

# Cell adjacency




**Adjacent cells on a Karnaugh map are those that differ by only one variable. Arrows point between adjacent cells.**

# Simplification through K-Maps

- KARNAUGH MAP sop minimization
  - the Karnaugh map is used for simplifying Boolean expressions to their minimum form.
  - A minimized SOP expression contains the fewest possible terms with the fewest possible variables per term.
  - Generally, a minimum SOP expression can be implemented with fewer logic gates than a standard expression.

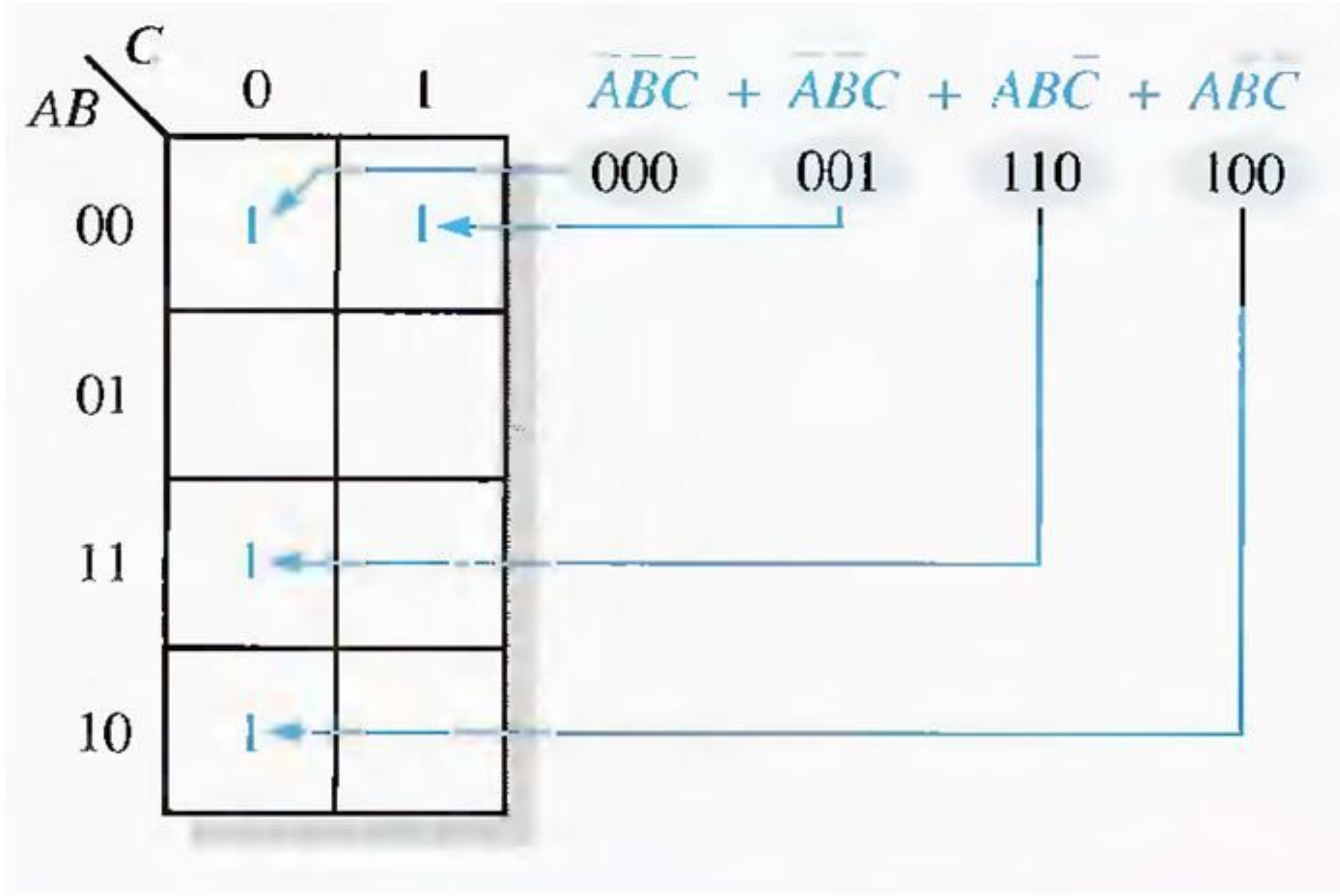
- Mapping a Standard SOP Expression
  - For an SOP expression in standard form, a 1 is placed on the Karnaugh map for each product term in the expression.
  - Each 1 is placed in a cell corresponding to the value of a product term. For example, for the product term AB'C, a 1 goes in the 101 cell on a 3-variable map.

# K-Maps

- When an SOP expression is completely mapped, there will be a number of 1's on the Karnaugh map equal to the number of product terms in the standard SOP expression.
- The cells that do not have a 1 are the cells for which the expression is O.
- Usually, when working with SOP expressions, the O's are left off the map.

- The following steps are used for mapping process.
  - **Step 1-** Determine the binary value of each product term in the standard SOP expression. After some practice, you can usually do the evaluation of terms mentally.
  - **Step 2.** As each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same value as the product term.

# Mapping entries in K-Map



# Mapping entries in K-Map

- Map the following standard SOP expression on a Karnaugh map:

**A′B′C + A′BC′ + ABC′ + ABC**

  - Solution:

**A′B′C + A′BC′ + ABC′ + ABC**

0 0 1 + 0 1 0 + 1 1 0 + 1 1 1

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |

# Self Assessment

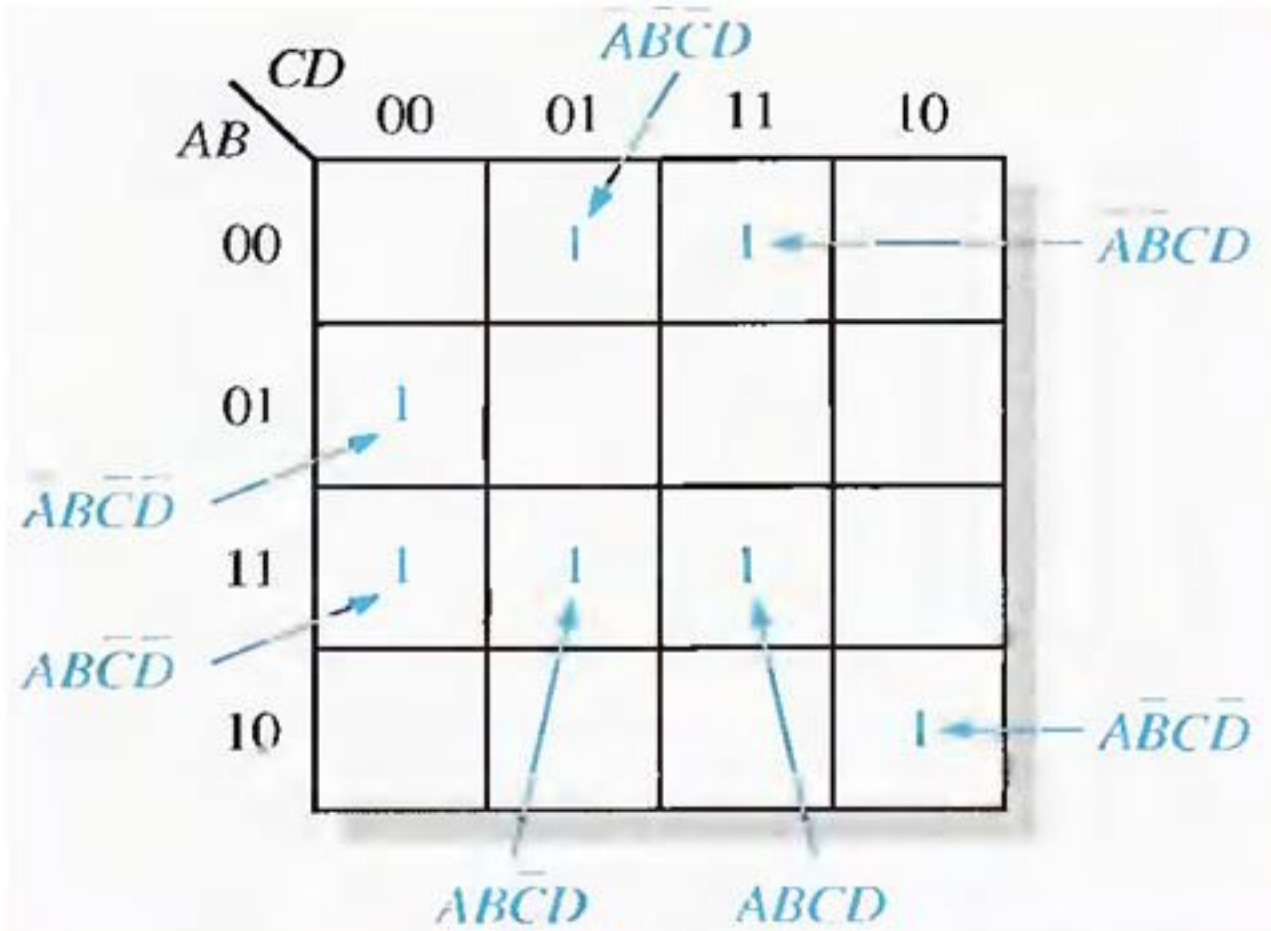
- **Problem:**
  - Map the standard SOP expression

    **A′ BC + AB′ C + AB′ C′**

    on a Karnaugh map just in 3 minutes, hurry up.

# 4-Variable K-Map

- Map the following standard SOP expression on a Karnaugh map:
  - **A′ B′ CD + A′ BC′ D′ + ABC′ D + ABCD + ABC′ D′ + A′ B′ C′ D + AB′ CD′**
- **Solution:**



# Self Assessment

- **Problem:**
  - Map the following standard SOP expression on a Karnaugh map:

**A′ BCD′ + ABCD′ + ABC′ D′ + ABCD**

**Map the above expression on a K-Map in just 2 Minutes**, Hurry Up.

# Mapping a Nonstandard SOP Expression

- A Boolean expression must first be in standard form before you use a Karnaugh map.
- If an expression is not in standard form, then it must be converted to standard form.
- It can also be done through numerical expansion.
- Numerical expansion is probably the most efficient approach.

# Mapping a Nonstandard SOP Expression

- Numerical Expansion of a Nonstandard Product Term:
  - Recall that a nonstandard product term has one or more missing variables.
  - Assume that one of the product term in a certain 3-variable SOP expression is AB′.
  - First, write the binary value of the two variables and attach a 0 for the missing variable C′: 100.
  - Next, write the binary value of the two variables and attach a 1 for the missing variable C: 101

# Mapping a Nonstandard SOP Expression

- As another example, assume that one of the product terms in a 3-variable expression is B.
- This term can be expanded numerically to standard form as follows.
- Write the binary value of the variable; then attach all possible values for the missing variables A and C as follows:

| $B$ |
|---|
| 010 |
| 011 |
| 110 |
| 111 |

**Note:**
**The four resulting binary numbers are the values of the standard SOP terms are A′ BC′ , A′ BC, ABC′, and ABC.**

# Example

- Map the following SOP expression on a Karnaugh map:

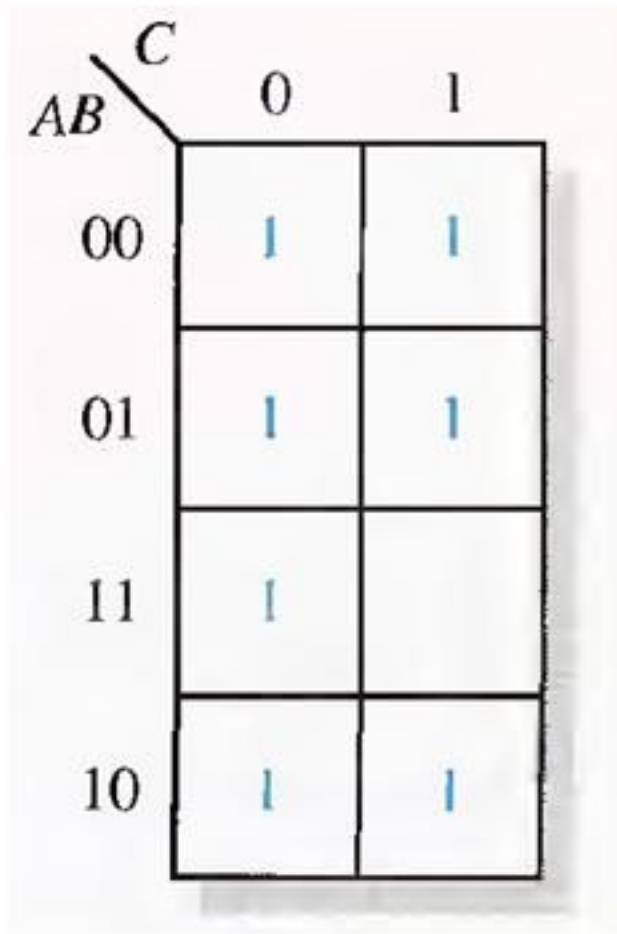
**A′ + AB′ + ABC′**

## Solution

- The first term is missing two variables, the second term is missing one variable, and the third term is standard.
- Numberical expansion of given expression can be done as follow,

# Mapping Non-standart SOP onto K-Map

**Now map each of the resulting binary values by placing a I in the appropriate cell of the 3-variable Karnaugh map**




| $\overline{A}$ | $+ A\overline{B}$ | $+ AB\overline{C}$ |
|---|---|---|
| 000 | 100 | 110 |
| 001 | 101 | |
| 010 | | |
| 011 | | |

**Problem**

Map the SOP expression

**BC + A′ C′**

on a Karnaugh map.

# Related Problem

- Map the following SOP expression on a Karnaugh map:

  **B′ C′ + AB′ + ABC′ + AB′ CD′ + A′ B′ C′ D + AB′ CD**

## Solution

- The SOP expression is obviously not in standard form because each product term does not have four variables.
- The first and second terms are both missing two variables, the third term is missing one variable, and the rest of the terms are standard.

# Example cont…

- First expand the terms by including all combinations of the missing variables numerically as follows:

| $\bar{B}\bar{C}$ | $A\bar{B}$ | + $AB\bar{C}$ | + $A\bar{B}C\bar{D}$ | + $\bar{A}\bar{B}\bar{C}D$ | + $A\bar{B}CD$ |
|---|---|---|---|---|---|
| 0000 | 1000 | 1100 | 1010 | 0001 | 1011 |
| 0001 | 1001 | 1101 | | | |
| 1000 | 1010 | | | | |
| 1001 | 1011 | | | | |

1. **Map each of the resulting binary values by placing a 1 in the appropriate cell of the 4- variable Karnaugh map.**
2. **Notice that some of the values in the expanded expression are redundant.**

# Example cont…

- After mapping the expression the resulting 4 veriable k map look like this,

**Related Problem:**

Map the expression

**A + C′ D + ACD′ + A′ BCD′**

on a Karnaugh map?

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | | |
| 01 | | | | |
| 11 | 1 | 1 | | |
| 10 | 1 | 1 | 1 | 1 |

- Your all the lectures notes are available at,

**Jadoon956.wordpress.com**


thanks
for the
tolerance