



# Number System

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**CIT LAHORE**

# Common Number Systems

<b>System</b>	<b>Base</b>	<b>Symbols</b>	<b>Used by humans?</b>	<b>Used in computers?</b>
<b>Decimal</b>	<b>10</b>	<b>0, 1, ... 9</b>	<b>Yes</b>	<b>No</b>
<b>Binary</b>	<b>2</b>	<b>0, 1</b>	<b>No</b>	<b>Yes</b>
<b>Octal</b>	<b>8</b>	<b>0, 1, ... 7</b>	<b>No</b>	<b>No</b>
<b>Hexa-decimal</b>	<b>16</b>	<b>0, 1, ... 9, A, B, ... F</b>	<b>No</b>	<b>Yes</b>

# Quantities/Counting (1 of 3)

<b>Decimal</b>	<b>Binary</b>	<b>Octal</b>	<b>Hexa- decimal</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
<b>2</b>	<b>10</b>	<b>2</b>	<b>2</b>
<b>3</b>	<b>11</b>	<b>3</b>	<b>3</b>
<b>4</b>	<b>100</b>	<b>4</b>	<b>4</b>
<b>5</b>	<b>101</b>	<b>5</b>	<b>5</b>
<b>6</b>	<b>110</b>	<b>6</b>	<b>6</b>
<b>7</b>	<b>111</b>	<b>7</b>	<b>7</b>

# Quantities/Counting (2 of 3)

<b>Decimal</b>	<b>Binary</b>	<b>Octal</b>	<b>Hexa- decimal</b>
<b>8</b>	<b>1000</b>	<b>10</b>	<b>8</b>
<b>9</b>	<b>1001</b>	<b>11</b>	<b>9</b>
<b>10</b>	<b>1010</b>	<b>12</b>	<b>A</b>
<b>11</b>	<b>1011</b>	<b>13</b>	<b>B</b>
<b>12</b>	<b>1100</b>	<b>14</b>	<b>C</b>
<b>13</b>	<b>1101</b>	<b>15</b>	<b>D</b>
<b>14</b>	<b>1110</b>	<b>16</b>	<b>E</b>
<b>15</b>	<b>1111</b>	<b>17</b>	<b>F</b>

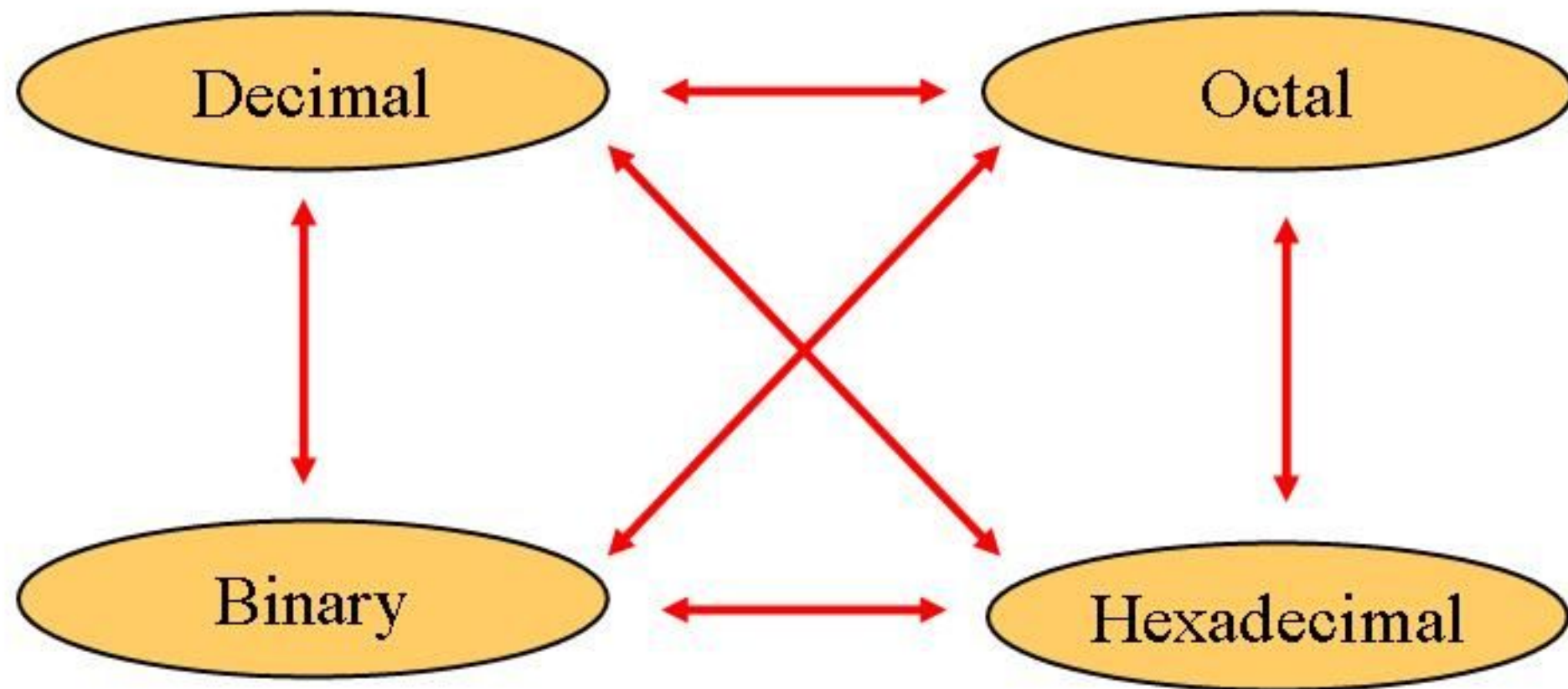
# Quantities/Counting (3 of 3)

<b>Decimal</b>	<b>Binary</b>	<b>Octal</b>	<b>Hexa- decimal</b>
<b>16</b>	<b>10000</b>	<b>20</b>	<b>10</b>
<b>17</b>	<b>10001</b>	<b>21</b>	<b>11</b>
<b>18</b>	<b>10010</b>	<b>22</b>	<b>12</b>
<b>19</b>	<b>10011</b>	<b>23</b>	<b>13</b>
<b>20</b>	<b>10100</b>	<b>24</b>	<b>14</b>
<b>21</b>	<b>10101</b>	<b>25</b>	<b>15</b>
<b>22</b>	<b>10110</b>	<b>26</b>	<b>16</b>
<b>23</b>	<b>10111</b>	<b>27</b>	<b>17</b>

Etc.

# Conversion Among Bases

- The possibilities:



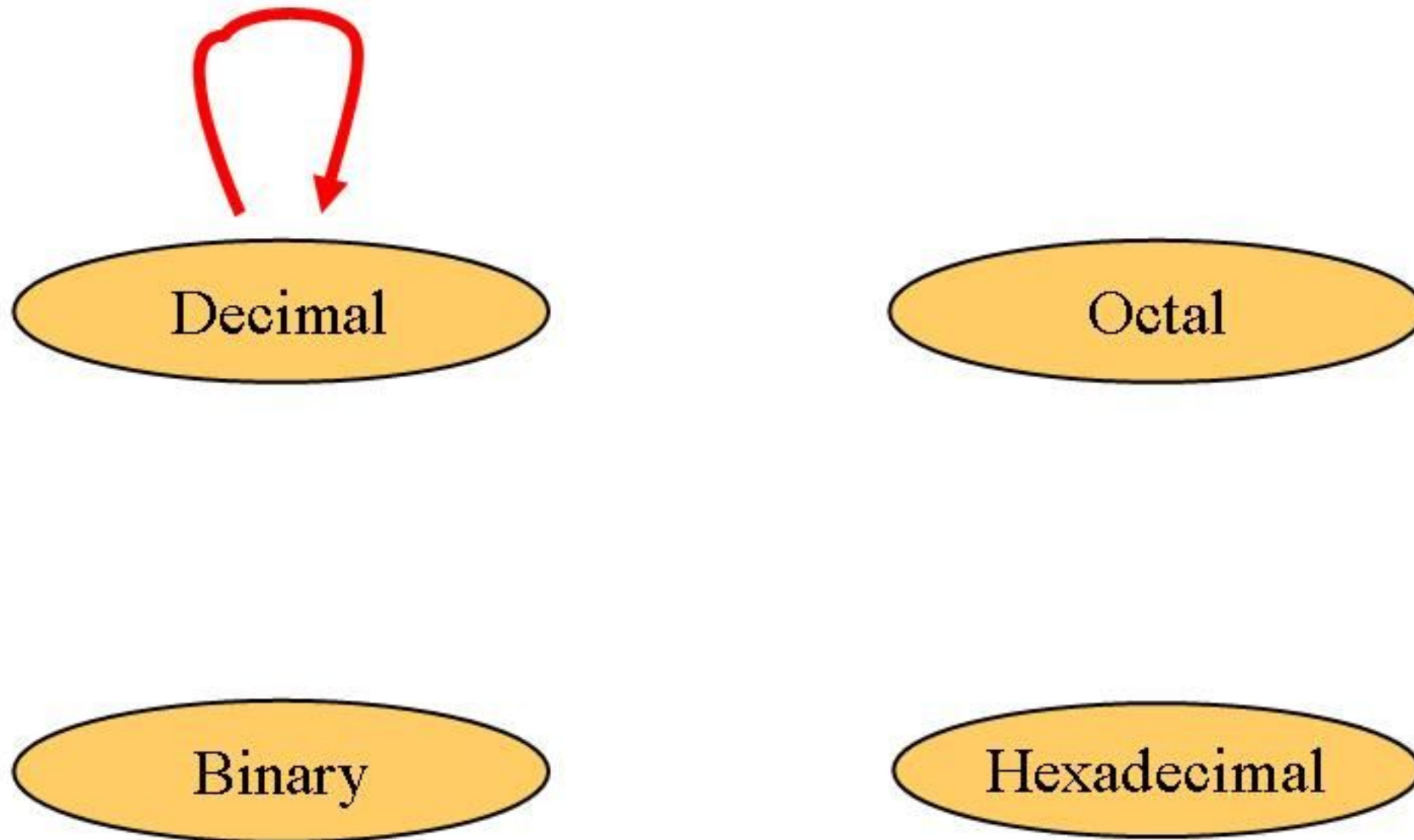
# Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$



Base

# Decimal to Decimal (just for fun)



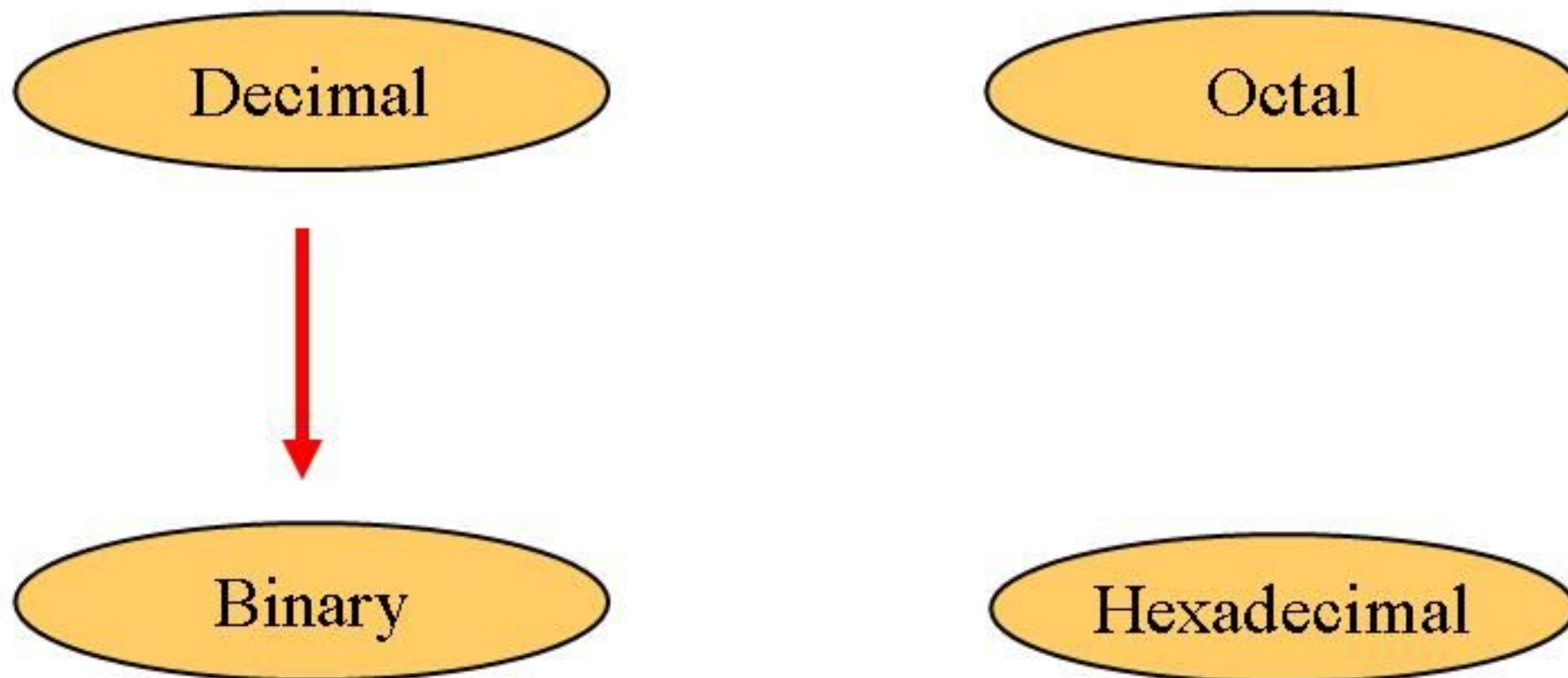


$$125_{10} \Rightarrow \begin{array}{r} 5 \times 10^0 \\ 2 \times 10^1 \\ 1 \times 10^2 \end{array} = \begin{array}{r} 5 \\ 20 \\ 100 \\ \hline 125 \end{array}$$

Weight

Base

# Decimal to Binary



## ■ Technique

- Divide by two, keep track of the remainder
- First remainder is bit 0 (LSB, least-significant bit)
- Second remainder is bit 1
- Etc.

# Example

$$125_{10} = ?_2$$

$$\begin{array}{r|l} 2 & 125 \\ \hline 2 & 62 \\ \hline 2 & 31 \\ \hline 2 & 15 \\ \hline 2 & 7 \\ \hline 2 & 3 \\ \hline & 1 \end{array}$$

1  
0  
1  
1  
1  
1  
1

$$125_{10} = 1111101_2$$

# Decimal to Binary conversion

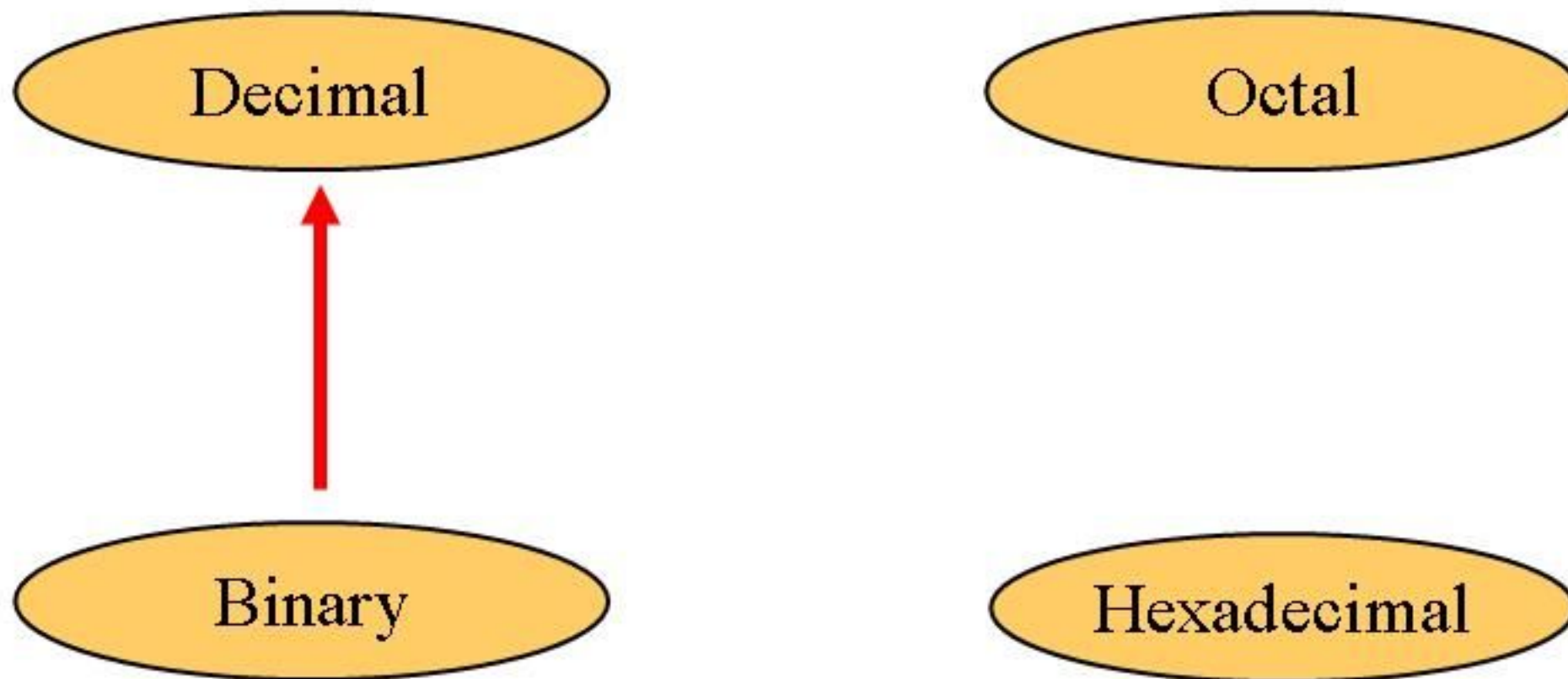
Convert  $(0.6875)_{10}$  to binary.

	<u>Integer</u>		<u>Fraction</u>
$0.6875 \times 2 =$	1	+	0.3750
$0.3750 \times 2 =$	0	+	0.7500
$0.7500 \times 2 =$	1	+	0.5000
$0.5000 \times 2 =$	1	+	0.0000

Answer:  $(0.6875)_{10} = (0.1011)_2$

$(41.6875)_{10} = (101001.1011)_2$

# Binary to Decimal



## ■ Technique

- Multiply each bit by  $2^n$ , where  $n$  is the “weight” of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

# Example

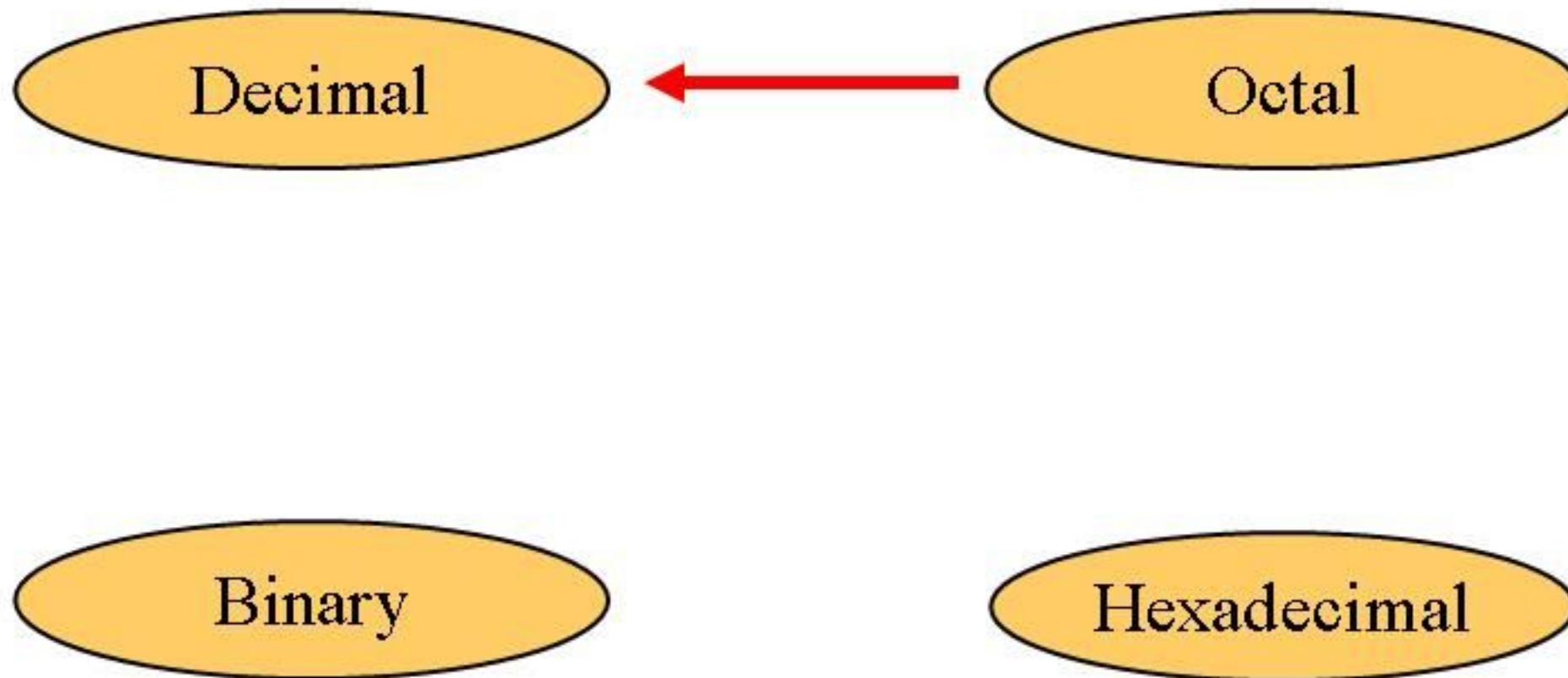
Bit "0"

$101011_2 \Rightarrow$

1	x	$2^0$	=	1
1	x	$2^1$	=	2
0	x	$2^2$	=	0
1	x	$2^3$	=	8
0	x	$2^4$	=	0
1	x	$2^5$	=	32
				<hr/>
				$43_{10}$



# Octal to Decimal



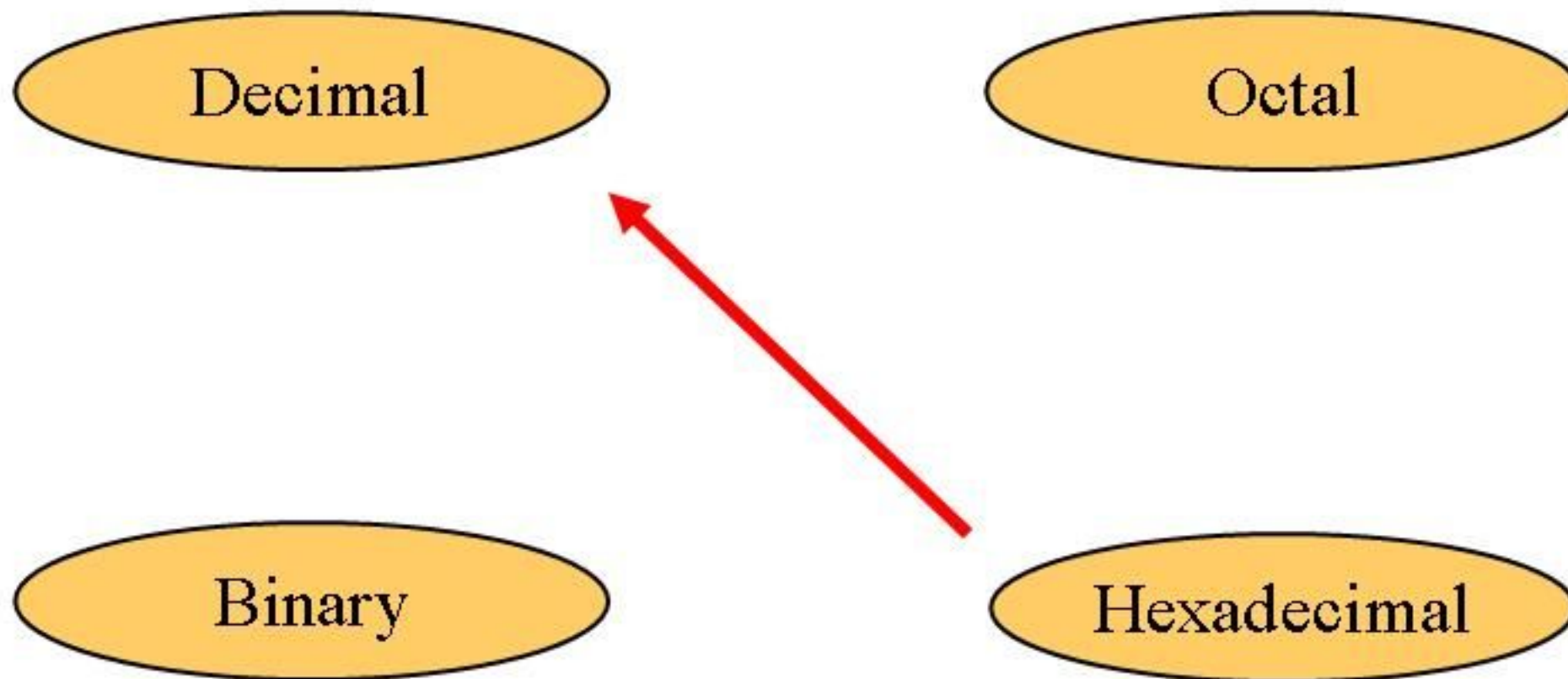
## ■ Technique

- Multiply each bit by  $8^n$ , where  $n$  is the “weight” of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

# Example

$$\begin{aligned} 724_8 &\Rightarrow 4 \times 8^0 = 4 \\ &\quad 2 \times 8^1 = 16 \\ &\quad 7 \times 8^2 = \underline{448} \\ &\quad \quad \quad 468_{10} \end{aligned}$$

# Hexadecimal to Decimal



# Hexadecimal to Decimal

## ■ Technique

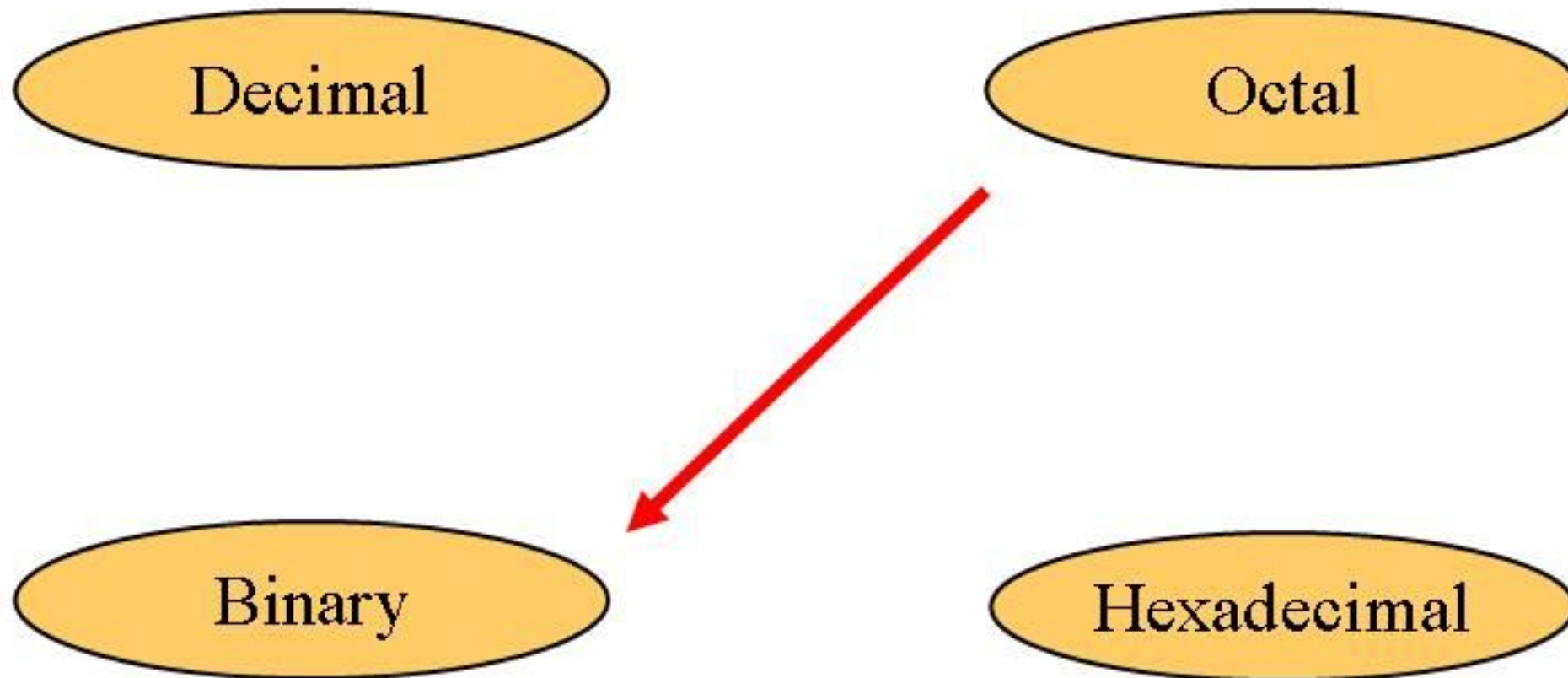
- Multiply each bit by  $16^n$ , where  $n$  is the “weight” of the bit.
- The weight is the position of the bit, starting from 0 on the right.
- Add the results.



# Example

$$\begin{aligned} \text{ABC}_{16} &\Rightarrow \text{C} \times 16^0 = 12 \times 1 = 12 \\ &\quad \text{B} \times 16^1 = 11 \times 16 = 176 \\ &\quad \text{A} \times 16^2 = 10 \times 256 = \underline{2560} \\ &\qquad\qquad\qquad 2748_{10} \end{aligned}$$

# Octal to Binary



- **Technique**
  - Convert each octal digit to a 3-bit equivalent binary representation



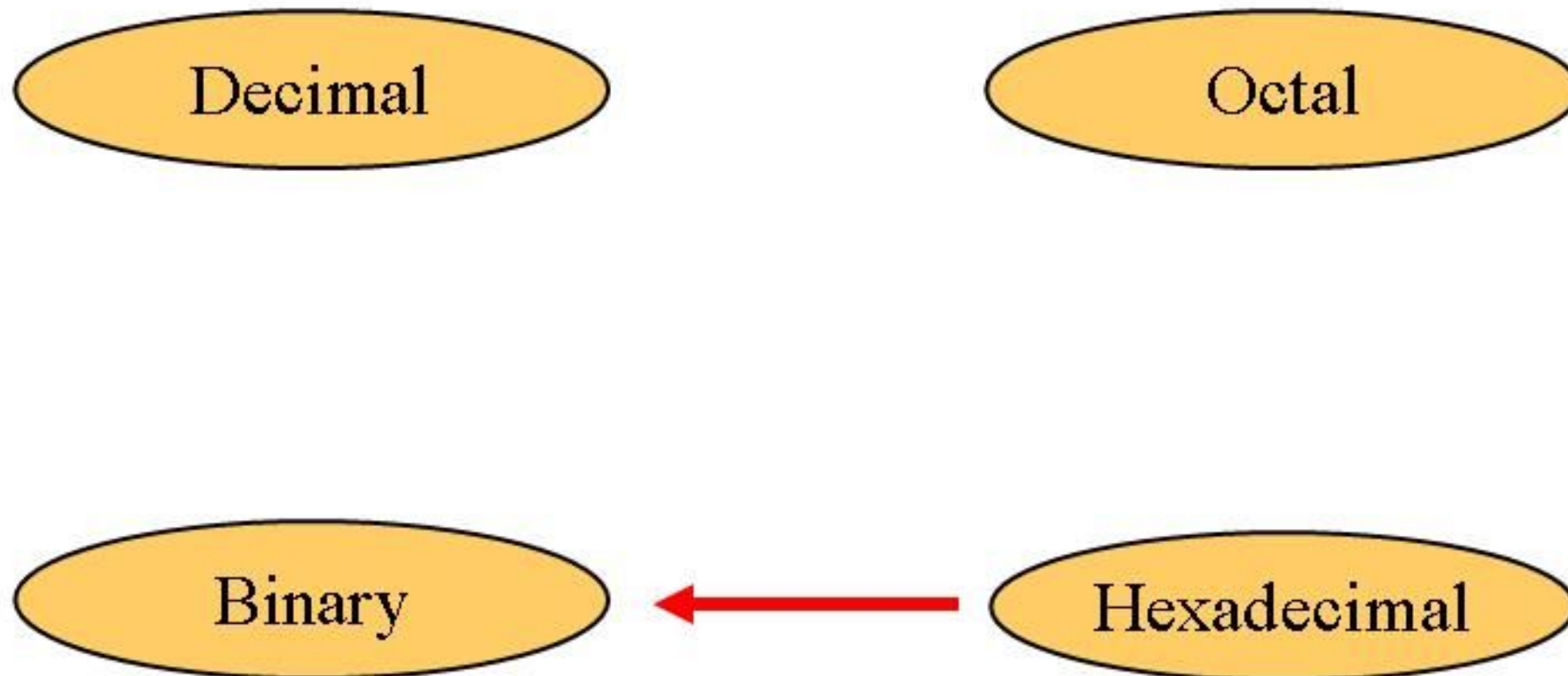
# Example

$$705_8 = ?_2$$

7	0	5
↓	↓	↓
111	000	101

$$705_8 = 111000101_2$$

# Hexadecimal to Binary



# Hexadecimal to Binary

- **Technique**
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation.



$$10AF_{16} = ?_2$$

1  
↓

0  
↓

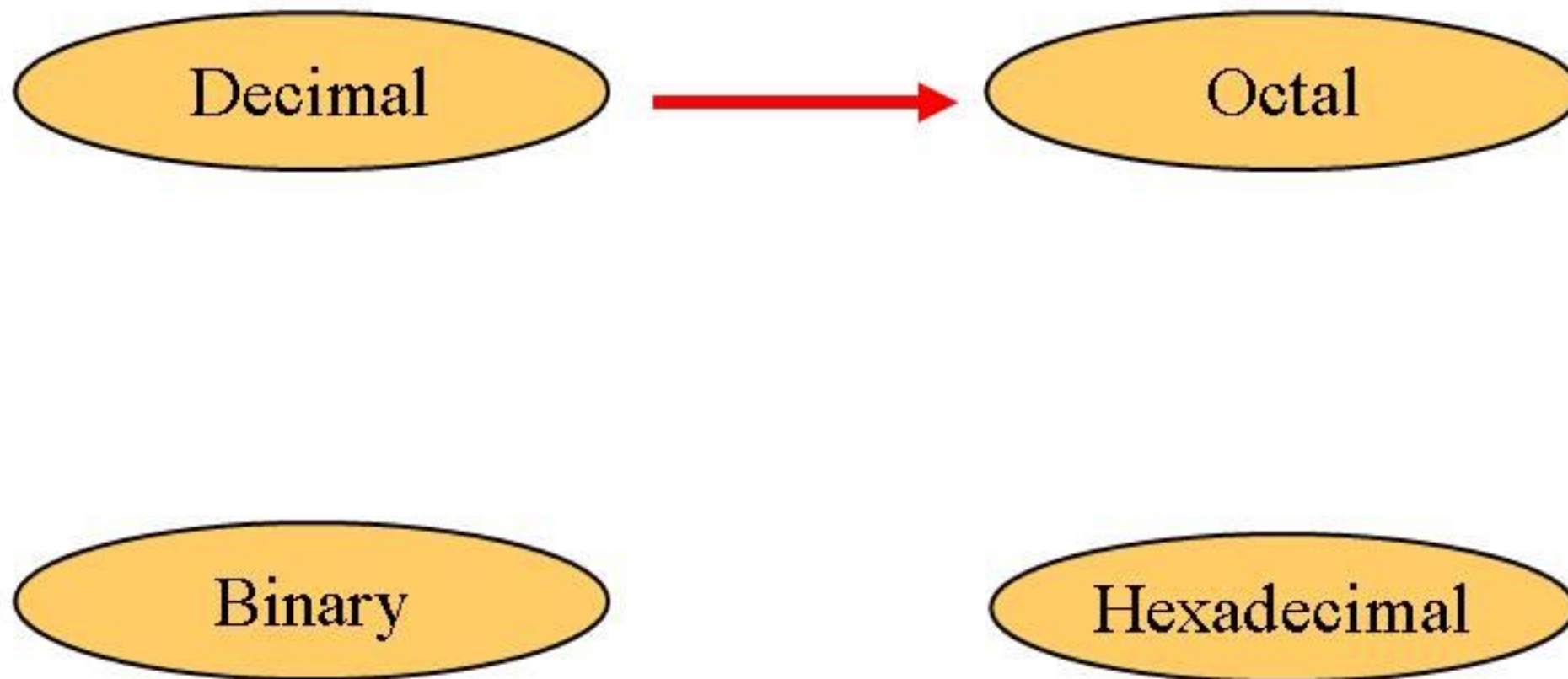
A  
↓

F  
↓

0001 0000 1010 1111

$$10AF_{16} = 0001000010101111_2$$

# Decimal to Octal



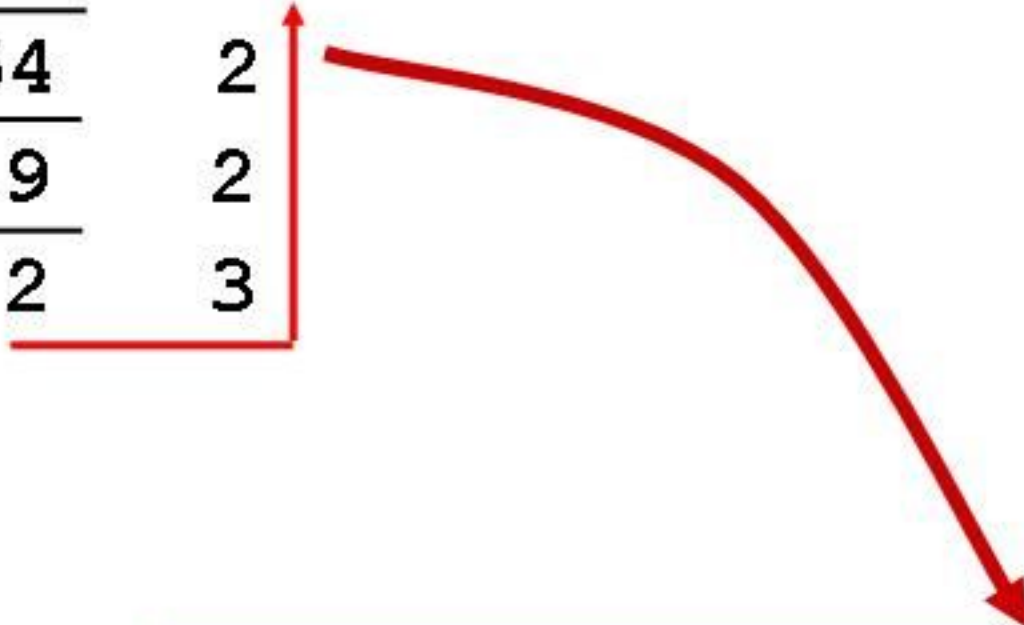
- **Technique**
  - Divide by 8
  - Keep track of the remainder

# Example

$$1234_{10} = ?_8$$

$$\begin{array}{r|l} 8 & 1234 \\ \hline 8 & 154 \\ \hline 8 & 19 \\ \hline & 2 \end{array}$$

2  
2  
3



$$1234_{10} = 2322_8$$

# Decimal to Octal Conversion

Convert  $(0.513)_{10}$  to octal.

$$0.513 \times 8 = 4.104$$

$$0.104 \times 8 = 0.832$$

$$0.832 \times 8 = 6.656$$

$$0.656 \times 8 = 5.248$$

$$0.248 \times 8 = 1.984$$

$$0.984 \times 8 = 7.872$$

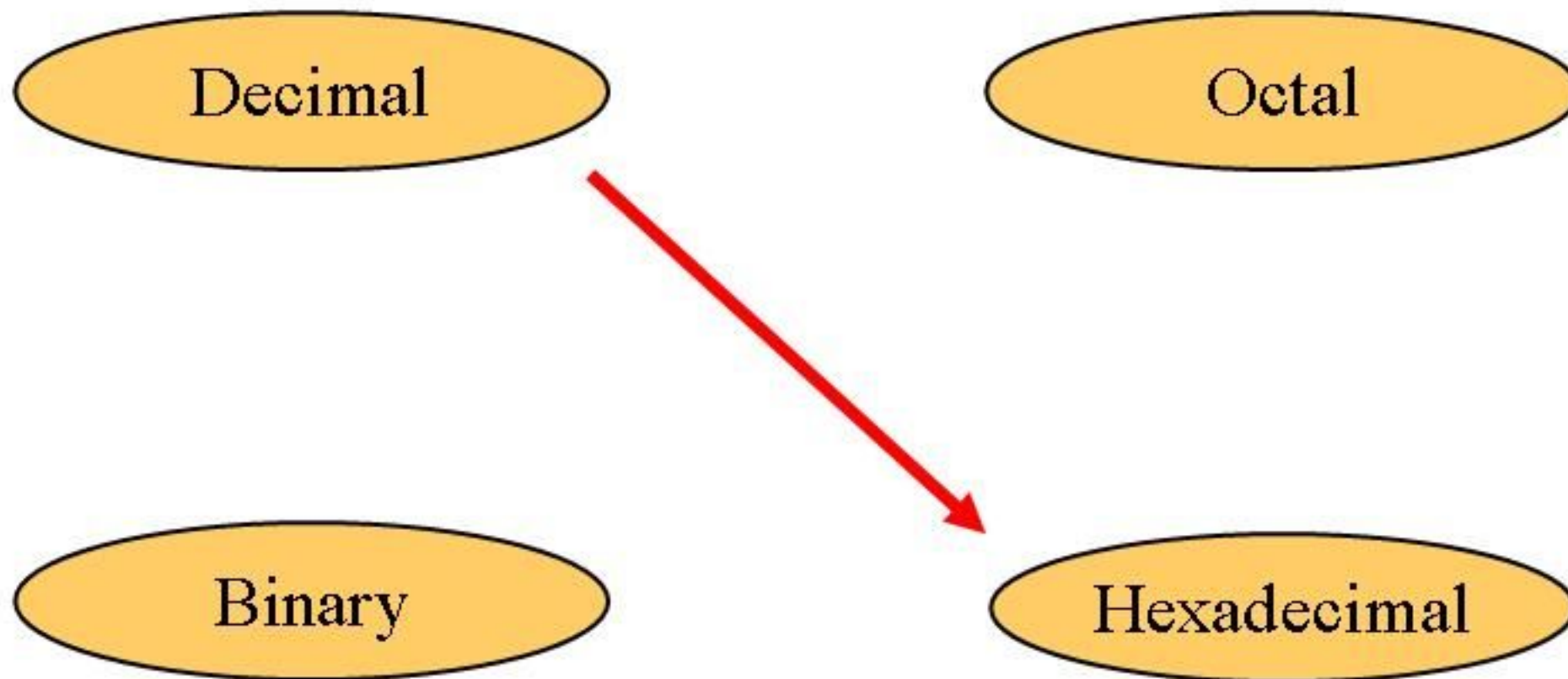
**Answer**  $(0.513)_{10} = (0.406517 \dots)_8$

$$(153.513)_{10} = (231.406517)_8$$





# Decimal to Hexadecimal



# Decimal to Hexadecimal

- **Technique**
  - Divide by 16
  - Keep track of the remainder



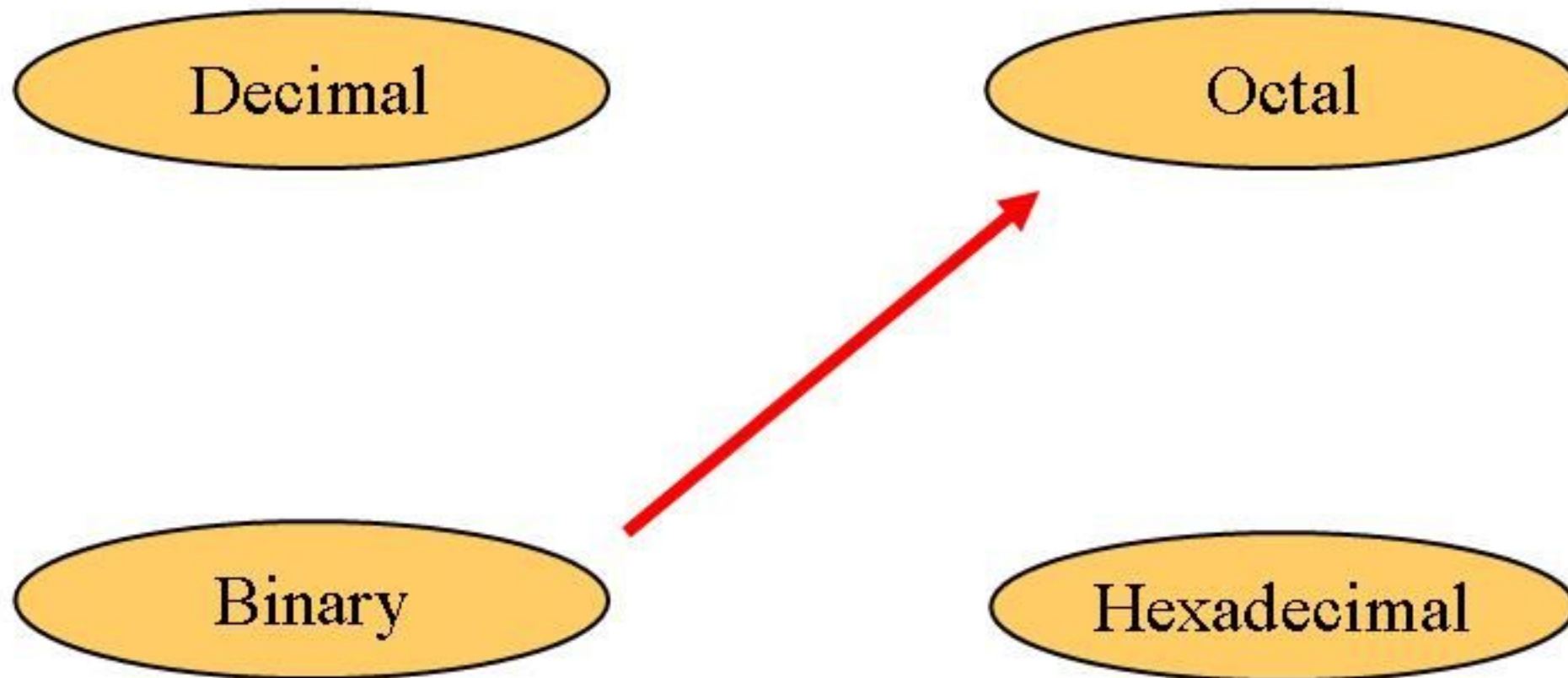
# Example

$$1234_{10} = ?_{16}$$

16	1234	
16	77	2
	4	13 = D

$$1234_{10} = 4D2_{16}$$

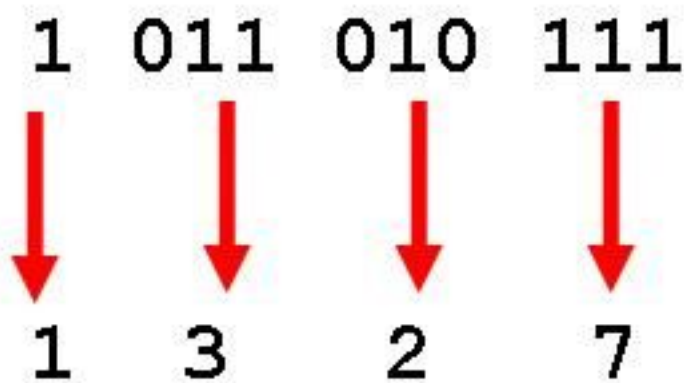
# Binary to Octal



- **Technique**
  - Group bits in threes, starting on right
  - Convert to octal digits

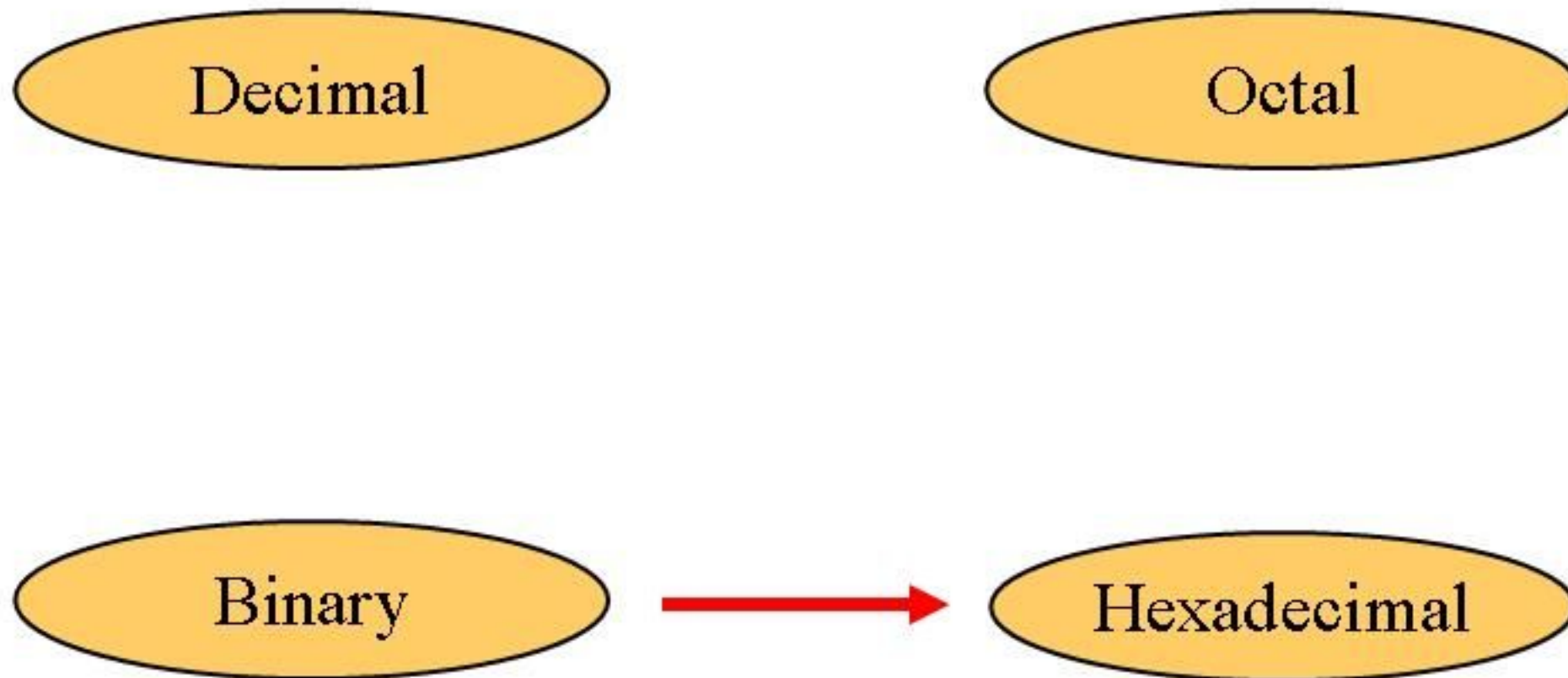
# Example

$$1011010111_2 = ?_8$$



$$1011010111_2 = 1327_8$$

# Binary to Hexadecimal



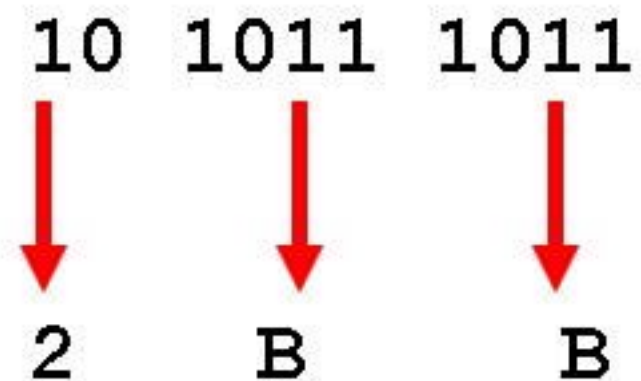
# Binary to Hexadecimal

- **Technique**
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits



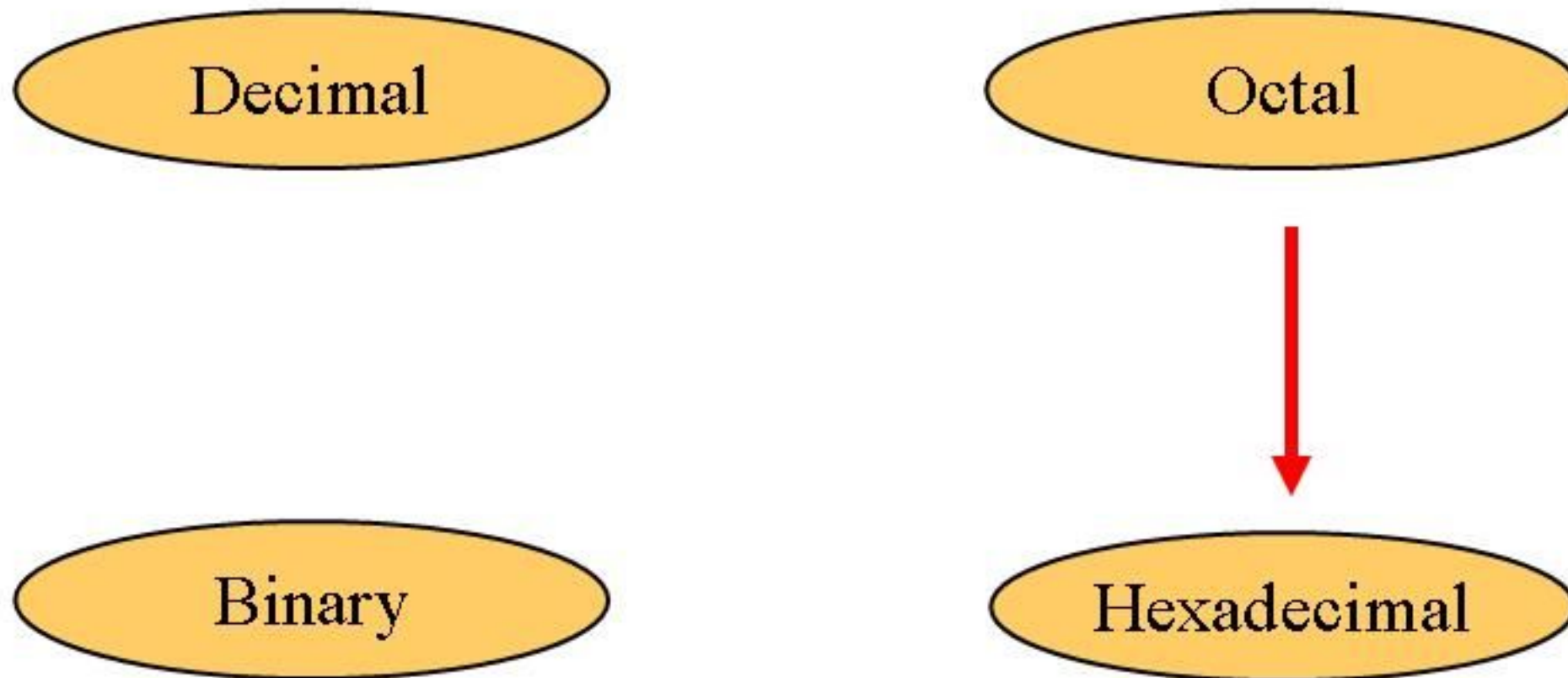
# Example

$$1010111011_2 = ?_{16}$$



$$1010111011_2 = 2BB_{16}$$

# Octal to Hexadecimal

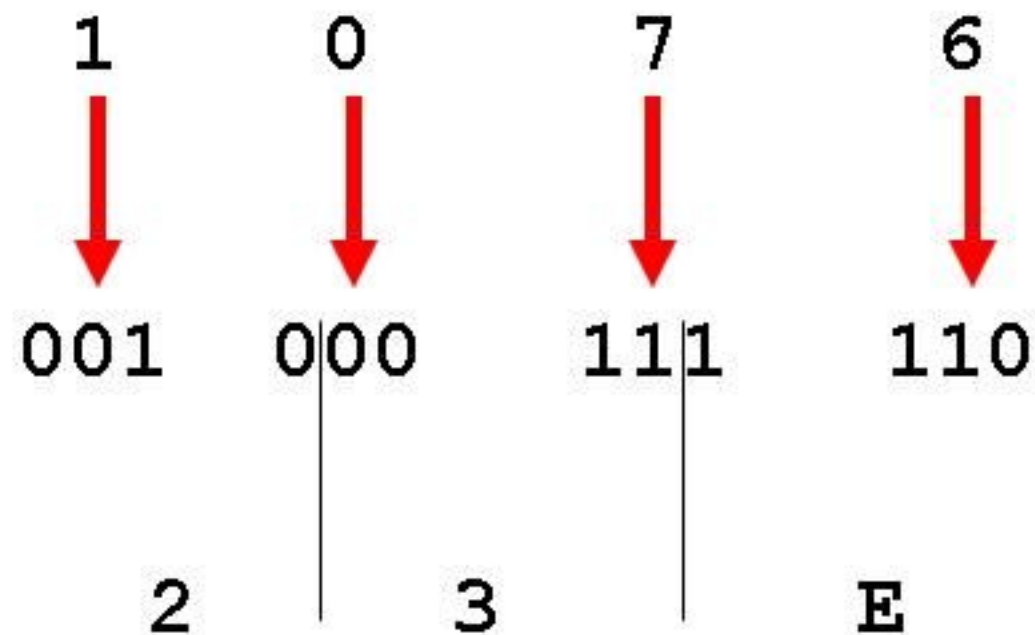


# Octal to Hexadecimal

- **Technique**
  - Use binary as an intermediary

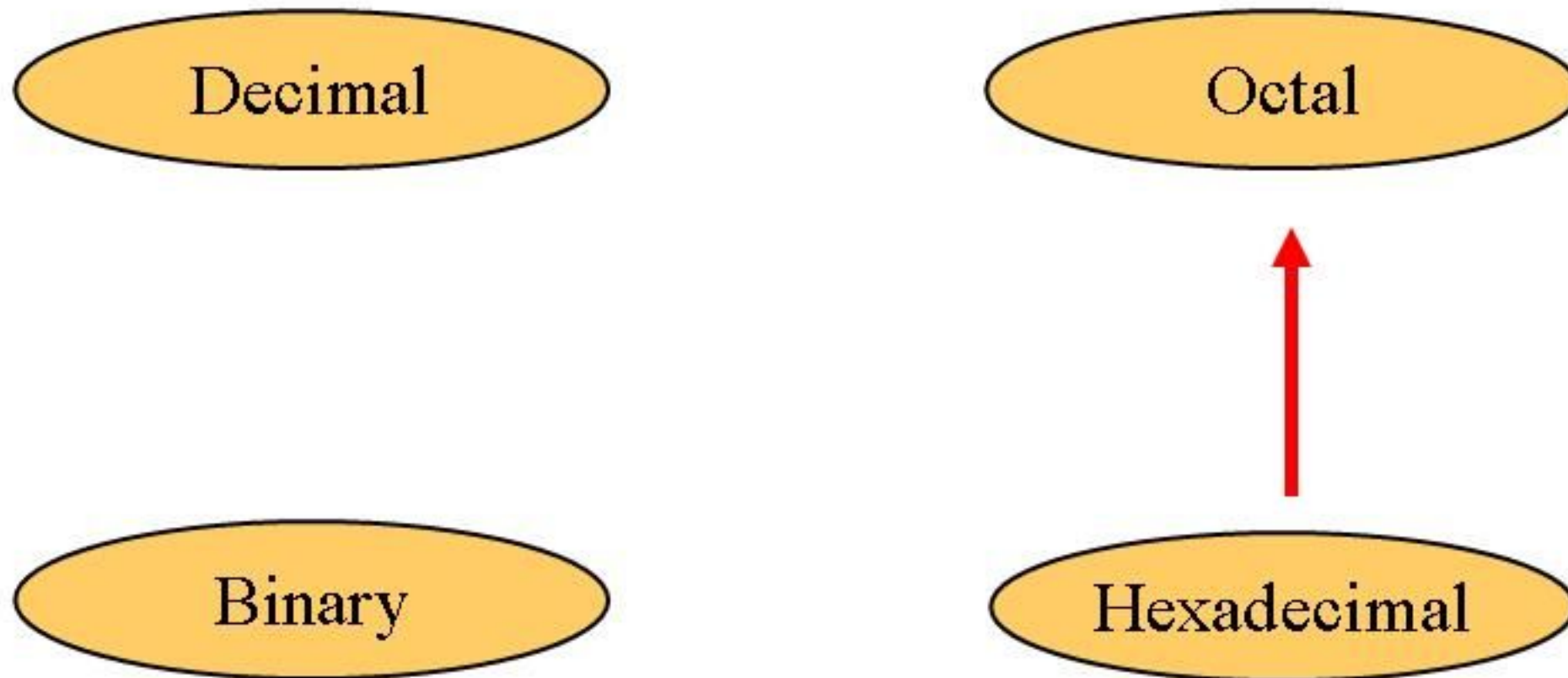
# Example

$$1076_8 = ?_{16}$$



$$1076_8 = 23E_{16}$$

# Hexadecimal to Octal

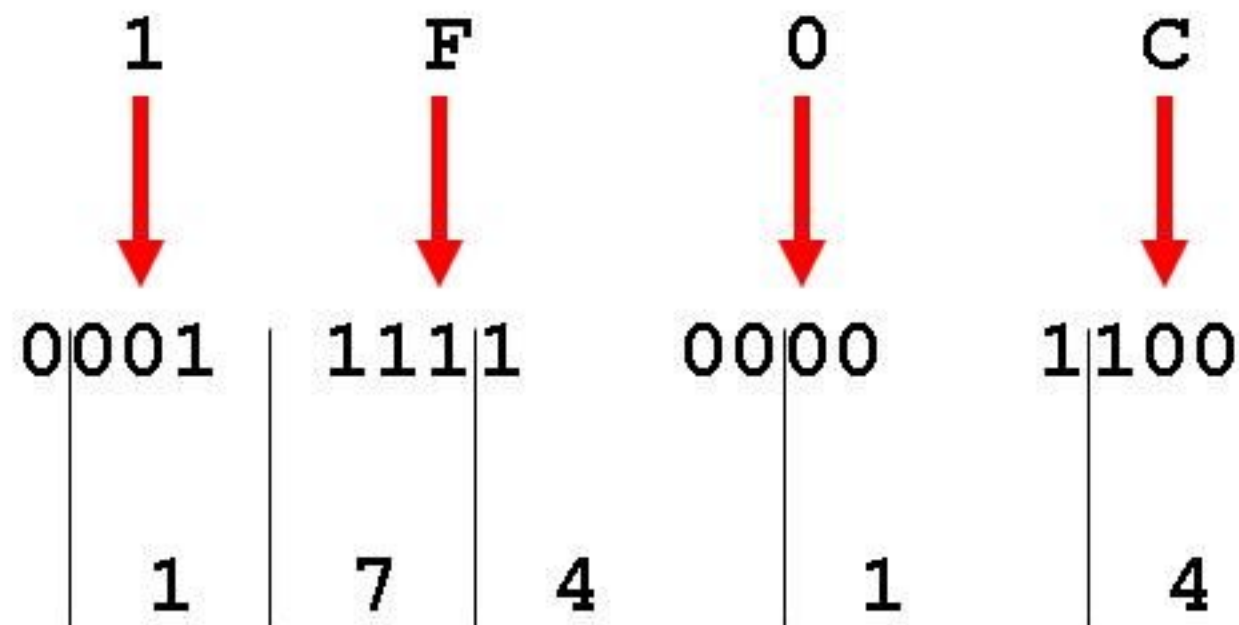


# Hexadecimal to Octal

- **Technique**
  - Use binary as an intermediary

# Example

$$1FOC_{16} = ?_8$$



$$1FOC_{16} = 17414_8$$

# Exercise – Convert ...

Decimal	Binary	Octal	Hexa- decimal
33			
	1110101		
		703	
			1AF



# Exercise – Convert ...

## Answer

Decimal	Binary	Octal	Hexa- decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF



# Common Powers (1 of 2)

## ■ Base 10

Power	Preface	Symbol	Value
$10^{-12}$	pico	p	.00000000000001
$10^{-9}$	nano	n	.0000000001
$10^{-6}$	micro	$\mu$	.000001
$10^{-3}$	milli	m	.001
$10^3$	kilo	k	1000
$10^6$	mega	M	1000000
$10^9$	giga	G	1000000000
$10^{12}$	tera	T	1000000000000

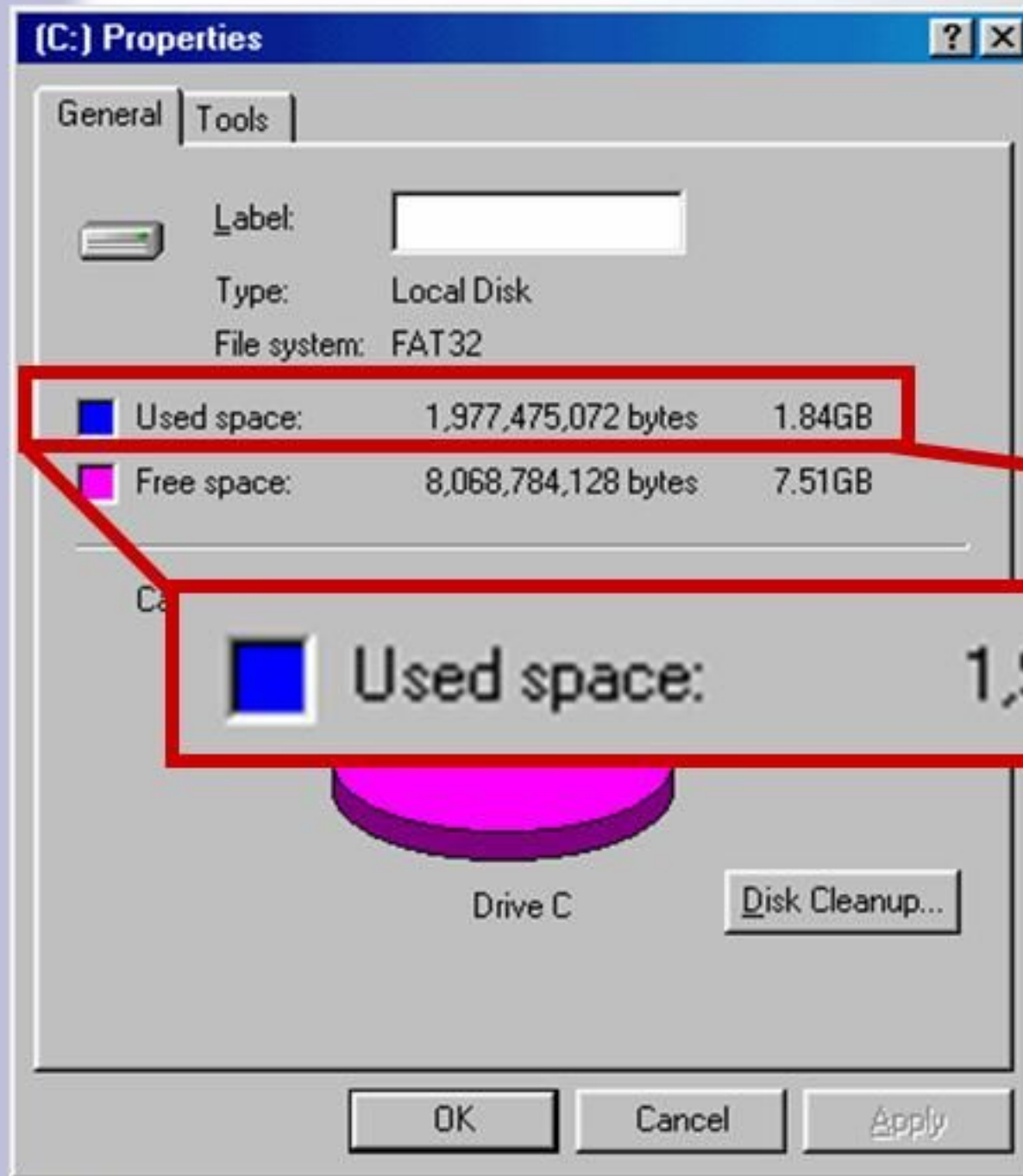
# Common Powers (2 of 2)

## ■ Base 2

Power	Preface	Symbol	Value
$2^{10}$	kilo	k	1024
$2^{20}$	mega	M	1048576
$2^{30}$	Giga	G	1073741824

- What is the value of “k”, “M”, and “G”?
- In computing, particularly w.r.t. memory, the base-2 interpretation generally applies

# Example



In the lab...

1. Double click on My Computer
2. Right click on C:
3. Click on Properties

$$/ 2^{30} =$$

## Review – multiplying powers

- For common bases, add powers

$$a^b \times a^c = a^{b+c}$$

$$2^6 \times 2^{10} = 2^{16} = 65,536$$

or...

$$2^6 \times 2^{10} = 64 \times 2^{10} = 64k$$

# Binary Addition (1 of 2)

- Two 1-bit values

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

“two”

# Binary Addition (2 of 2)

- Two  $n$ -bit values
  - Add individual bits
  - Propagate carries
  - E.g.,

$$\begin{array}{r} \phantom{+} \overset{1}{1}0101 \\ + \phantom{+} 11001 \\ \hline 101110 \end{array}$$

$$\begin{array}{r} \phantom{+} 21 \\ + \phantom{+} 25 \\ \hline 46 \end{array}$$

# Multiplication (1 of 3)

- Decimal (just for fun)

$$\begin{array}{r} 35 \\ \times 105 \\ \hline 175 \\ 000 \\ 35 \\ \hline 3675 \end{array}$$



## Multiplication (2 of 3)

- Binary, two 1-bit values

A	B	$A \times B$
0	0	0
0	1	0
1	0	0
1	1	1

## Multiplication (3 of 3)

- Binary, two  $n$ -bit values
  - As with decimal values
  - E.g.,

$$\begin{array}{r} 1110 \\ \times 1011 \\ \hline 1110 \\ 1110 \\ 0000 \\ 1110 \\ \hline 10011010 \end{array}$$

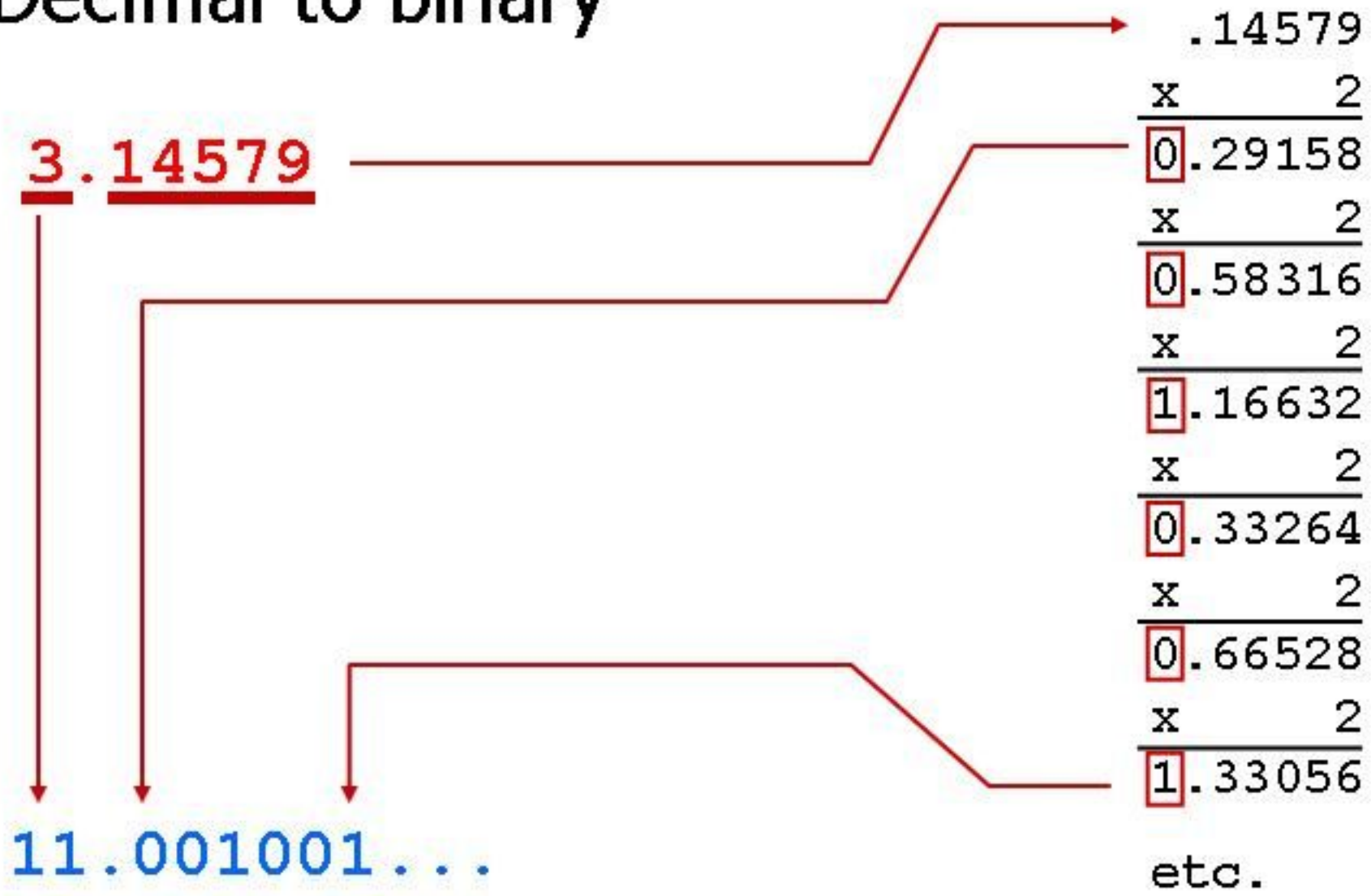
- Decimal to decimal (just for fun)

$$\begin{array}{r} 3.14 \Rightarrow 4 \times 10^{-2} = 0.04 \\ 1 \times 10^{-1} = 0.1 \\ 3 \times 10^0 = \frac{3}{\phantom{0.04}} \\ \hline 3.14 \end{array}$$

- Binary to decimal

$$\begin{array}{r} 10.1011 \Rightarrow \\ 1 \times 2^{-4} = 0.0625 \\ 1 \times 2^{-3} = 0.125 \\ 0 \times 2^{-2} = 0.0 \\ 1 \times 2^{-1} = 0.5 \\ 0 \times 2^0 = 0.0 \\ 1 \times 2^1 = 2.0 \\ \hline 2.6875 \end{array}$$

## ■ Decimal to binary



# Exercise – Convert ...

Decimal	Binary	Octal	Hexa- decimal
29.8			
	101.1101		
		3.07	
			C.82

# Exercise – Convert ...

Answer

Decimal	Binary	Octal	Hexa- decimal
29.8	11101.110011...	35.63...	1D.CC...
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82





thanks  
for the  
**tolerance**